DP Mathematics: analysis and approaches HL

Syllabus overview

This book covers the whole syllabus for the DP Mathematics: analysis and approaches HL course. Here is an overview of the syllabus content covered in each chapter.

1 From patterns to generalizations: sequences, series and proofs

Syllabus reference	Syllabus content
SL1.2*	Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.
SL1.3*	Geometric sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of geometric sequences. Applications.
SL1.4*	Financial applications of geometric sequences and series: compound interest and annual depreciation.
SL1.6	Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity.
SL1.8	Sum of infinite convergent geometric sequences.
SL1.9	The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$.
AHL1.10	Counting principles, including permutations and combinations. Extension of the binomial theorem to fractional and negative indices, ie $(a+b)^n$, $n \in \mathbb{Q}$).
AHL1.15	Proof by mathematical induction. Proof by contradiction. Use of a counterexample to show that a statement is not always true.

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2 Representing relationships: functions

Syllabus reference	Syllabus content
SL2.2*	Concept of a function, domain, range and graph. Function notation, for example $f(x)$, $v(t)$, $C(n)$. The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.
SL2.3*	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.4*	Linear correlation of bivariate data. Pearson's product−moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.
SL2.5	Composite functions. Identity function. Finding the inverse function $f^{-1}(x)$.
SL2.6	The quadratic function $f(x) = ax^2 + bx + c$: its graph, y-intercept 0, c. Axis of symmetry. The form $f(x) = a(x - p)(x - q)$, x intercepts $(p, 0)$ and $(q, 0)$. The form $f(x) = a(x - h)^2 + k$, vertex (h, k).
SL2.7	Solution of quadratic equations and inequalities. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.
SL2.8	The reciprocal function $f(x) = \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature. Rational functions of the form $f(x) = \frac{ax+b}{cx+d}$ and their graphs. Equations of vertical and horizontal asymptotes.
SL2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.

SL2.11	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x) - a$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = pf(x)$. Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$. Composite transformations.
AHL1.11	Partial fractions.
AHL2.12	Polynomial functions, their graphs and equations; zeros, roots and factors. The factor and remainder theorems. Sum and product of the roots of polynomial equations.
AHL2.13	Rational functions of the form: $f(x) = \frac{ax+b}{cx^2+dx+e}$ and $f(x) = \frac{ax^2+bx+c}{dx+e}$
AHL2.14	Odd and even functions. Finding the inverse function, $f^{-1}(x)$, including domain restriction. Self-inverse functions.
AHL2.15	Solutions of $g(x) \geq f(x)$, both graphically and analytically.
AHL2.16	The graphs of the functions $y = \|f(x)\|$ and $y = f(\|x\|)$, $y = \frac{1}{f(x)}$, $y = f(ax + b)$, $y = \|f(x)\|^2$. Solution of modulus equations and inequalities.

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3 Expanding the number system: complex numbers

Syllabus reference	Syllabus content
SL2.6	The quadratic function $f(x) = ax^2 + bx + c$: its graph, y-intercept 0, c. Axis of symmetry. The form $f(x) = a(x - p)(x - q)$, x-intercepts $(p, 0)$ and $(q, 0)$. The form $f(x) = a(x - h)^2 + k$, vertex (h, k).
SL2.7	Solution of quadratic equations and inequalities. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.
AHL1.12	Complex numbers: the number i, where $i^2 = -1$. Cartesian form $z = a + bi$; the terms real part, imaginary part, conjugate, modulus and argument. The complex plane.
AHL1.14	Complex conjugate roots of quadratic and polynomial equations with real coefficients. De Moivre's theorem and its extension to rational exponents. Powers and roots of complex numbers.
AHL1.16	Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.
AHL2.12	Polynomial functions, their graphs and equations; zeros, roots and factors. The factor and remainder theorems. Sum and product of the roots of polynomial equations.

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4 Measuring change: differentiation

Syllabus reference	Syllabus content
SL2.1*	Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients, m_1 and m_2 Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.
SL5.1*	Introduction to the concept of a limit. Derivative interpreted as gradient function and as rate of change.
SL5.2*	Increasing and decreasing functions. Graphical interpretation of $f'(x)>0, f'(x)=0, f'(x)<0$.
SL5.3*	Derivative of $f(x) = ax^n \ \ f'(x) = anx^{n-1}, n \in \mathbb{Z}$ The derivative of functions of the form $f(x) = ax^n\_bx^{n-1}$ . . . where all exponents are integers.
SL5.4*	Tangents and normals at a given point, and their equations.
SL5.6	Derivative of x^n ($n \in \mathbb{Q}$), sin x, cos x, e^x and ln x. Differentiation of a sum and a multiple of these functions. The chain rule for composite functions. The product and quotient rules.
SL5.7	The second derivative. Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''.
SL5.8	Local maximum and minimum points. Testing for maximum and minimum. Optimization. Points of inflexion with zero and non-zero gradients.
SL5.9	Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.
AHL5.12	Informal understanding of continuity and differentiability of a function at a point. Understanding of limits (convergence and divergence). Definition of derivative from first principles $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ Higher derivatives.
AHL5.14	Implicit differentiation. Related rates of change. Optimisation problems.

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5 Analysing data and quantifying randomness: statistics and probability

Syllabus reference	Syllabus content
SL4.1*	Concepts of population, sample, random sample, discrete and continuous data. Reliability of data sources and bias in sampling. Interpretation of outliers. Sampling techniques and their effectiveness.
SL4.2*	Presentation of data (discrete and continuous): frequency distributions (tables). Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). Production and understanding of box and whisker diagrams.
SL4.3*	Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion (interquartile range, standard deviation and variance). Effect of constant changes on the original data. Quartiles of discrete data.
SL4.4*	Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.
SL4.10	Equation of the regression line of x on y. Use of the equation for prediction purposes.

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6 Relationships in space: geometry and trigonometry

Syllabus reference	Syllabus content
SL3.1*	The distance between two points in three- dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.
SL3.2*	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2ab \sin C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Area of a triangle as $\frac{1}{2} ab \sin C$.
SL3.3*	Applications of right and non-right angled trigonometry, including Pythagoras' theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.
SL3.4	The circle: radian measure of angles; length of an arc; area of a sector.
SL3.5	Definition of $\tan\theta$ as $\frac{sin\theta}{\cos\theta}$.
SL3.6	Exact values of trigonometric ratios of 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and their multiples. Extension of the sine rule to the ambiguous case.
SL3.7	The circular functions sin x, cos x, and tan x; amplitude, their periodic nature, and their graphs Composite functions of the form $f(x) = a\sin(b(x+c)) + d$. Transformations. Real-life contexts.
SL3.8	Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in sin x, cos x, or tan x.
AHL3.9	Definition of the reciprocal trigonometric ratios $\sec\theta$, $\mathrm{cosec}\,\theta$ and $\cot\theta$.

	$1+\tan^2\theta=\sec^2\theta$, Pythagorean identities: $1+\cot^2\theta=\text{cosec}^2\theta$. The inverse functions $f(x)=\arcsin x$, $f(x)=\arccos x$, $f(x)=\arctan x$; their domains and ranges; their graphs.
AHL3.10	Compound angle identities. Double angle identity for tan.
AHL3.11	Relationships between trigonometric functions and the symmetry properties of their graphs.

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7 Generalizing relationships: exponents, logarithms and integration

Syllabus reference	Syllabus content
SL1.5*	Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.
SL1.7	Laws of exponents with rational exponents. Laws of logarithms. $\log_a xy = \log_a x + \log_a y$. $\log_a \frac{x}{y} = \log_a x - \log_a y$. $\log_a x^m = m\log_a x$ for $a, x, y > 0$. Change of base of a logarithm. $\log_a x = \frac{\log_b x}{\log_b a}$ for $a, b, x > 0$. Solving exponential equations, including using logarithms.
SL2.9	Exponential functions and their graphs: $f(x) = a^x, a > 0, f(x) = \mathrm{e}^x$. Logarithmic functions and their graphs: $f(x) = \log_a x, x > 0, f(x) = \ln x, x > 0$.
SL2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.
SL2.11	Definite integrals, including analytical approach. Areas of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x)$ can be positive or negative, without the use of technology. Areas between curves.
SL5.6	Derivative of x^n ($n \in \mathbb{Q}$), sin x, cos x, e^x and ln x. Differentiation of a sum and a multiple of these functions. The chain rule for composite functions. The product and quotient rules.
SL5.5*	Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \ldots$, where $n \in \mathbb{Z}$, $n \neq -1$ Anti-differentiation with a boundary condition to determine the constant term. Definite integrals using technology. Areas between a curve $y = f(x)$ and the

	x-axis, where $f(x) > 0$.
SL5.10	Indefinite integral of x^n $(x \in \mathbb{Q})$, sin x, cos x, $\frac{1}{x}$ and e^x. The composites of any of these with the linear function $ax + b$. Integration by inspection (reverse chain rule) or by substitution for expressions of the form: $\int kg'(x)f\big(g(x)\big)\mathrm{d}x$.
SL5.11	Definite integrals, including analytical approach. Areas of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x)$can be positive or negative, without the use of technology. Areas between curves.
AHL5.15	Derivatives of tan x, sec x, cosec x, cot x, a^x, $\log_a x$, arcsin x, arccos x, arctan x. Indefinite integrals of the derivatives of any of the above functions. The composites of any of these with a linear function. Use of partial fractions to rearrange the integrand.
AHL5.16	Integration by substitution. Integration by parts. Repeated integration by parts.

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8 Modelling change: more calculus

Syllabus reference	Syllabus content
SL5.9	Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.
SL5.11	Definite integrals, including analytical approach. Areas of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x)$ can be positive or negative, without the use of technology. Areas between curves.
AHL5.13	The evaluation of limits of the form $\lim_{x \to a} \frac{f(x)}{g(x)}$ and $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ using l'Hôpital's rule. Repeated use of l'Hôpital's rule.
AHL5.17	Area of the region enclosed by a curve and the y-axis in a given interval. Volumes of revolution about the x-axis or y-axis.
AHL5.18	First order differential equations. Numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$ using Euler's method. Variables separable. Homogeneous differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = f(\frac{y}{x})$ using the substitution $y = vx$. Solution of $y' + P(x)y = Q(x)$, using the integrating factor.
AHL5.19	Maclaurin series to obtain expansions for e^x, sin x, cos x, ln(1 + x), $(1 + x)^p$, $p \in \mathbb{Q}$. Use of simple substitution, products, integration and differentiation to obtain other series. Maclaurin series developed from differential equations.

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9 Modelling 3D space: vectors

Syllabus reference	Syllabus content
AHL3.12	Concept of a vector; position vectors; displacement vectors. Representation of vectors using directed line segments. Base vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$. Components of a vector: $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \boldsymbol{v_1 i} + \boldsymbol{v_2 j} + \boldsymbol{v_3 k}$. Algebraic and geometric approaches to the following: the sum and difference of two vectors the zero vector **0**, the vector $-\boldsymbol{v}$ multiplication by a scalar, $\boldsymbol{kv}$, parallel vectors magnitude of a vector, $\lvert\boldsymbol{v}\rvert$; unit vectors, $\frac{\boldsymbol{v}}{\lvert\boldsymbol{v}\rvert}$ position vectors $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$ displacement vector: $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ Proof of geometrical properties using vectors.
AHL3.13	The definition of the scalar product of two vectors. The angle between two vectors. Perpendicular vectors; parallel vectors.
AHL3.14	Vector equation of a line in two and three dimensions: The angle between two lines. Simple applications to kinematics.
AHL3.15	Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection.
AHL3.16	The definition of the vector product of two vectors. Properties of the vector product. Geometric interpretation of $\lvert\boldsymbol{v} \times \boldsymbol{w}\rvert$
AHL3.17	Vector equations of a plane: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$, where $\boldsymbol{b}$ and $\boldsymbol{c}$ are non-parallel vectors within the plane. $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$, where $\boldsymbol{n}$ is a normal to the plane and $\boldsymbol{a}$ is the position vector of a point on the plane. Cartesian equation of a plane $ax + by + cz = d$.
AHL3.18	Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes.

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10 Equivalent systems of representations: more complex numbers

Syllabus reference	Syllabus content
AHL1.12	Complex numbers: the number i, where $i^2 = -1$. Cartesian form $z = a + bi$; the terms real part, imaginary part, conjugate, modulus and argument. The complex plane.
AHL1.13	Modulus–argument (polar) form: $z = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$. Euler form: $z = re^{i\theta}$ Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation.
AHL1.14	Complex conjugate roots of quadratic and polynomial equations with real coefficients. De Moivre's theorem and its extension to rational exponents. Powers and roots of complex numbers.

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11 Valid comparisons and informed decisions: probability distributions

Syllabus reference	Syllabus content
SL4.2*	Presentation of data (discrete and continuous): frequency distributions (tables). Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). Production and understanding of box and whisker diagrams.
SL4.5*	Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is $\mathrm{P}(A)=\frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Expected number of occurrences.
SL4.6*	Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities. Combined events: $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$. Mutually exclusive events: $\mathrm{P}(A\cap B)=0$ Conditional probability: $\mathrm{P}(AB)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ Independent events: $\mathrm{P}(A\cap B)=\mathrm{P}(A)\mathrm{P}(B)$.
SL4.7*	Concept of discrete random variables and their probability distributions. Expected value (mean), for discrete data. Applications.
SL4.8*	Binomial distribution. Mean and variance of the binomial distribution.
SL4.9*	The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation. Normal probability calculations. Inverse normal calculations
SL4.11	Formal definition and use of the formulae: $\mathrm{P}(A\|B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ for conditional probabilities, and $\mathrm{P}(A\|B)=\mathrm{P}(A)=\mathrm{P}(A\|B')$ for independent events.
SL4.12	Standardization of normal variables (z-values). Inverse normal calculations where mean and standard deviation are unknown.
AHL4.13	Use of Bayes' theorem for a maximum of three events.

AHL4.14	Variance of a discrete random variable. Continuous random variables and their probability density functions. Mode and median of continuous random variables. Mean, variance and standard deviation of both discrete and continuous random variables. The effect of linear transformations of X.
AHL1.10	Counting principles, including permutations and combinations. Extension of the binomial theorem to fractional and negative indices, ie $(a+b)^n$, $n \in \mathbb{Q}$.

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